Bifurcation diagram of a mapping

```
1  from scipy import *
2  from pylab import *
```

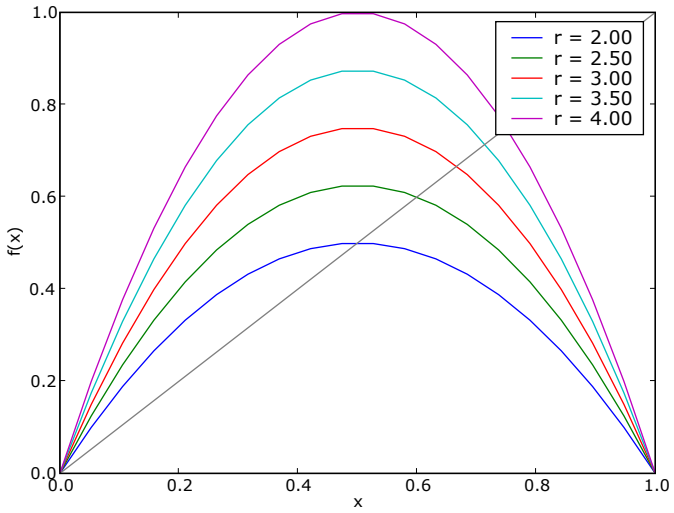
We are interested in the long term behavior of a sequence created by a the iteration of map.

The logistic map

```
10 f = lambda x,r : r * x * ( 1 -x )
```

The logistic map is parametrised by "r"

```
12 x = linspace( 0, 1, 20 )
13 rlist = linspace( 2, 4, 5 )
14 hold(True)
15 for r in rlist:  plot(x, f(x,r), label = 'r = %.2f' % r)
16 legend()
17 xlabel('x')
18 ylabel('f(x)')
19 plot(x,x,color=(0.5,0.5,0.5), label = ' ')
20 show()
```



Behavior of the sequence

The sequence is created by iteration of the map over an initial value:

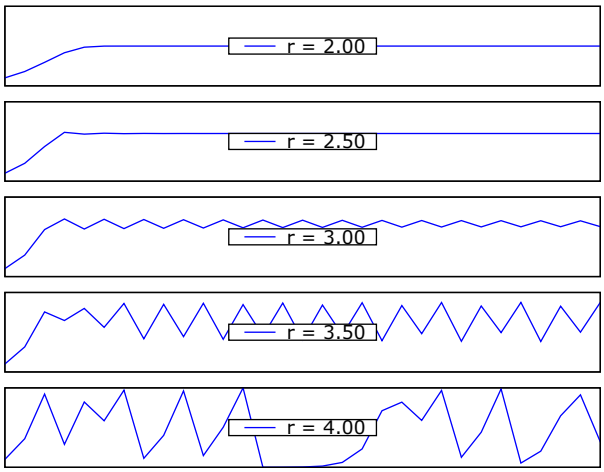
```
24 X = [0.1, ]
25 for i in arange(0,9):   X += [f(X[-1],2)]
26 print array(X)
```

```
[ 0.1          0.18         0.2952       0.41611392   0.48592625   0.49960386
  0.49999969   0.5          0.5          0.5          ]
```

The sequence converges to a stable fixed point if it has one, but can also oscillated between different unstable fixed points, or have no stable long term behavior, exibiting chaos.

```
30 X = [ 0.1 * ones_like(rlist), ]
31 for i in arange(0,30):   X += [ f(X[ -1], rlist) ]
32 X = vstack(X)
33 figure()
34 for i, r in enumerate(rlist):
35     subplot( rlist.size, 1, i+1)
36     plot( X[ :, i], label = 'r = %.2f' % r)
```

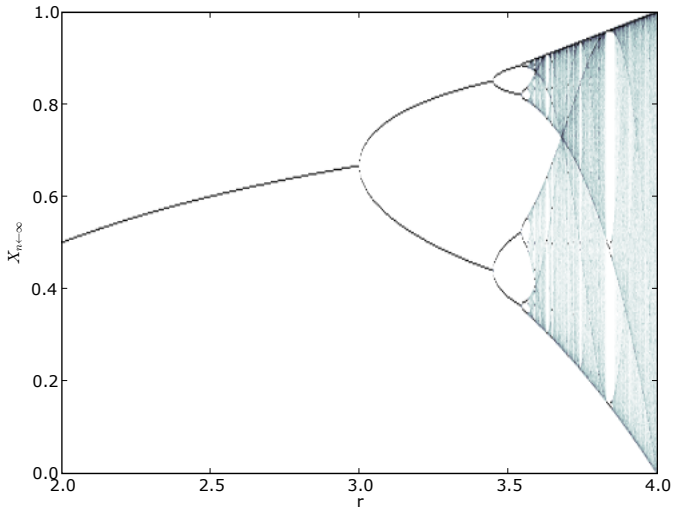
```
    ylim ( 0, 1)
    yticks('')
    xticks('')
    legend( loc = 10 )
show()
```



Bifurcation diagram

To study the lont term behavior of the sequence we can plot the values it visit after many iterations, as a function of the parameter

```
rlist = linspace( 2, 4, 800)
X = [ 0.5 * ones_like(rlist), ]
for i in arange(0,10000):   X += [ f(X[ -1], rlist) , ]
X = hsplit( vstack(X[-2000:]), rlist.size)
from scipy import stats
H = map( lambda Z : stats.histogram( Z, defaultlimits=(0,1), numbins=300 )[0],X)
H = map( lambda Z : 1-Z/Z.max(), H )
H = vstack(H)
figure()
imshow( rot90(H), aspect = 'auto' , extent = [2, 4, 0, 1])
bone()
xlabel('r')
ylabel(r'$X_{n \rightarrow \infty}$')
show()
```


1.0
0.8
0.6
0.4
0.2
0.0
$X_{n+\infty}$
2.0
2.5
3.0
3.5
4.0
r